# First Semester M.Tech. Degree Examination, December 2010
## Applied Mathematics

Time: 3 hrs. Max. Marks:100



**Note:** ***Answer any FIVE full questions.***

1 a. How do you convert a decimal number to a binary number? Find the binary form of the numbers 295 and 6/10. **(10 Marks)**

b. What is error and relative error and significant digit in numerical methods? **(04 Marks)**

c. The number 2.71828183 is approximated as 2.7183. Find the following : i) Error ii) Relative error iii) Number of significant digits of the approximation. **(06 Marks)**

2 a. What are vector norms and matrix norms? Find the matrix norms L1, L2 and Le of the matrix $A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$. **(07 Marks)**

b. Using Gauss – Jordan elimination method, solve the system of equations :
$2x - y + z = 4$ , $4x + 3y - z = 6$ , $3x + 2y + 2z = 15$. **(07 Marks)**

c. Use Gauss – Seidal iteration method to solve the system of equations.
$10x + y + z = 12$ , $2x + 10y + z = 13$ $2x + 2y + 10z = 14$ with initial approximation $y = 0, z = 0$. **(06 Marks)**

3 a. By using Given's method reduce the matrix
$A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{bmatrix}$ to tridiagonal form. **(08 Marks)**

b. Find the largest eigen value in modulus and the corresponding eigen vector of the matrix.
$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using power method, starting with the initial vector $[1, 1, 1]^T$. **(12 Marks)**

4 a. Consider the pressure (p), specific volume(v) relationship given by Vander Waals equation $p = \dfrac{RT}{v-b} - \dfrac{a}{v^2}$ . Determine the first and second order derivatives of p, $\dfrac{dp}{dv}$ and $\dfrac{dp^2}{dv^2}$, at $v = 0.05$ using backward and forward difference formulas. Given
R = Specific gas constant = 0.461889 kJ/kg-K ; T = Temperature in Kelvin = 623.15 ;
a = 1.7048 ; b = 0.0016895. **(10 Marks)**

b. Let f(x, y) be a two – dimensional function. Find the finite – difference approximation for the partial derivatives $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial^2 f}{\partial x^2}, \dfrac{\partial^2 f}{\partial y^2}$ and $\dfrac{\partial^2 f}{\partial x \partial y}$ at $(x_i, y_j)$. **(10 Marks)**

5 a. A plane area is bounded by a curve, the x –axis and two extreme ordinates. The area is divided into five figures by equidistant ordinates 2 cms apart, the heights of the ordinates being 21.65, 21.04, 20.35, 19.61, 18.75 and 17.80 cms respectively. Find the approximate value of the area using trapezoidal rule. **(06 Marks)**

b. Evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by dividing the interval into eight equal parts, using Simpson's 1/3 rule. **(08 Marks)**

c. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss – Legendre three point formula. What is the exact solution? **(06 Marks)**

6 a. State the Euler and modified Euler's methods of solving ordinary differential equations. Use modified Euler's method to find an approximate solution at the point 0.4 of the problem $\frac{dy}{dx} = -y$ ; $y(0) = 1$, with step length $h = 0.2$. **(08 Marks)**

b. State the Runge – Kutta fourth order formula. Solve the differential equation $\frac{dy}{dx} = xy$, $y(1) = 2$ with $h = 0.2$, using this formula. **(08 Marks)**

c. State the fourth order Adams predictor – corrector method to solve a linear differential equation. **(04 Marks)**

7 a. The temperature distribution in a rectangular fin, considering conduction and radiation heat transfers, is given by $\frac{d^2T}{dx^2} = \frac{\sigma \in P}{KA}(T^4 - T_\infty^4)$. Refer fig. Q7(a)

Where T = temperature , K = thermal conductivity , A = bd = cross – sectional area , P = 2(b + d) = perimeter , $T_\infty$ = surrounding temperature , σ = Stefan – Boltzman constant , ∈ = emissivity. The data are given by $k = 42\ W/m - {}^0k$ , $b = 0.5m$ , $d = 0.2m$ , $T_\infty = 500K$ , $\ell = 2m$ , $\in = 0.1$ , $\sigma = 5.7 \times 10^{-8}\ W/m^2\text{-}K^4$ , $T(x = 0) = 1000K$ and $T(x = \ell) = 350K$. Find the solution of this nonlinear boundary value problem using finite difference method. **(10 Marks)**

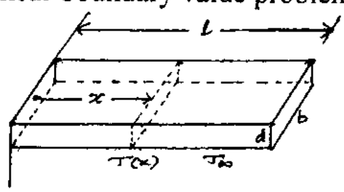



Fig.Q7(a) Rectangular Fin

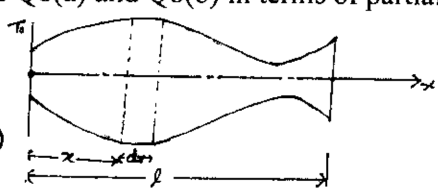
b. The differential equation governing the transverse deflection of a beam w(x), subjected to a distributed load, p(x) as shown in figure Q7(b), is given by $\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) = p(x)$,

where E = Young's modulus and I = area moment of inertia of the beam. Formulate the boundary value problem for a uniform beam, i) fixed at both ends and ii) simply supported at both ends. **(10 Marks)**


Fig.Q7(b) Beam under distributed load

8 a. Derive the equation governing the temperature distribution in a one – dimensional fin as shown in figures Q8(a) and Q8(b) in terms of partial derivatives. **(08 Marks)**

Fig.Q8 (a)




One dimensional fin

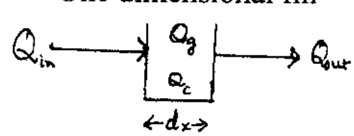



Fig.Q8 (b)

b. Derive the finite – difference equations for solving the Poisson equation $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = f(x,y)$ over a rectangular region of size 10″× 12″ using Δx = 5″ and Δy = 6″ , with the following boundary conditions : **(12 Marks)**

$\frac{\partial\phi}{\partial x} - \phi = 2$ at x = 0 , $\frac{\partial\phi}{\partial x} - 2\phi = -1$ at y = 0 , φ = 3 at x = 10 in , φ = 4 at y = 12 in.

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